On the Nature of Turbulence in a Stratified Fluid. Part I: The Energetics of Mixing

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ABSTRACT

The definition of the flux Richardson number R_f is generalized to be the ratio of the turbulent buoyancy flux b to the net turbulent mechanical energy m available from all sources. For mechanically energized turbulence where turbulent kinetic energy is used to sustain an upward buoyancy flux ($b > 0$), it is shown the magnitude of R_f is quantitatively determined by the location of the event in the $\mathrm{Fr_T}$–$\mathrm{Re_T}$ diagram, where $\mathrm{Fr_T}$ and $\mathrm{Re_T}$ are the local instantaneous overturn Froude number and Reynolds number. In this parameter space, the value of R_f varies between 0 and 0.20 for a fluid with Prandtl number greater than one, and between 0 and 0.15 for a fluid with a Prandtl number less than one.

For turbulence sustained by a negative buoyancy flux ($b < 0$), such as penetrative convection in a cooling surface layer, it is shown that the flux Richardson number R_f is a function of depth below the surface; R_f^{-1} varies between 0.55 at the surface and $-\infty$ towards the base of the surface layer where the buoyancy flux vanishes. This result may again be interpreted in terms of location in the $\mathrm{Fr_T}$–$\mathrm{Re_T}$ diagram.

Finally, it is shown that once the value of R_f is known the vertical buoyancy flux may be evaluated directly without recourse to a turbulence model.

1. Introduction

The nature of the turbulence in a mechanically generated turbulent flow leading to a mixing event in a stratified fluid is most clearly visualized by an examination of the Froude and Reynolds numbers characterizing the turbulent motion and the ambient stratification (Gibson 1980, 1982, 1986, 1987a, 1987b; Imberger and Boashash 1986; Luketina and Imberger 1989). If N denotes the buoyancy frequency of the background stratification, u the rms velocity of the turbulent motions, and L_C the centered displacement scale, a measure of the scale of the observed overturns or inversions actually registered in the density profile (see Imberger and Boashash 1986), then the energy bearing eddies are characterized by the overturn Froude number

$$\mathrm{Fr_T} = \frac{u}{NL_C}. \qquad (1)$$

Since L_C is the scale of the most energetic overturns, Luketina and Imberger (1989) postulated that the rms velocity scale u is given by

$$u \sim (\epsilon L_C)^{1/3} \qquad (2)$$

where ϵ is the rate of dissipation of turbulent kinetic energy.

This assumption can be tested with the laboratory results in a stratified water tunnel facility reported by Stillinger et al. (1983), Itsweire et al. (1986) (hereafter denoted by SHV and IHV, respectively) with turbulence generated by a grid in salt stratified water; by Rohr et al. (1988, hereafter denoted by RIHV) with turbulence generated by both a grid and a mean shear in salt stratified water; and with data from the recent work by Lienhard and Van Atta (1990, hereafter denoted by LV) with turbulence generated by a grid in a temperature stratified air tunnel. The energy cascade process described by (2) should be independent of Pr and this is indeed the case as can be seen in Fig. 1 which shows an excellent correlation between the rms velocity u and $(\epsilon L_T)^{1/3}$. The quantity L_T is measured in the above experiments from the rms density fluctuations and the known background density gradient and differs from L_C by a scale factor very close to one (Luketina, personal communication). The implication of the relation in (2) is that the small-scale motions, constituting the high wavenumber components of the turbulence, keep pace in their development with the energy input at larger scales (see also Broadwell and Breindentahl 1982) during the energetic stages of the grid generated turbulence.

Substituting (2) into (1) yields alternative definitions of the Froude number as (Luketina and Imberger 1989)

$$\mathrm{Fr_T} = \frac{u}{NL_C} = \left(\frac{\Gamma}{N}\right) = \left(\frac{\epsilon}{N^3 L_C^2}\right)^{1/3} = \left(\frac{L_R}{L_C}\right)^{2/3} \qquad (3)$$

where the rate of strain of the large-scale fluctuations $\Gamma = (u/L_C)$, and the Ozmidov scale $L_R = (\epsilon/N^3)^{1/2}$.

If we note that $N^2 L_C^2$ is the potential energy per unit

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