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On the Nature of Turbulence in a Stratified Fluid. Part II: Application to Lakes

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ABSTRACT

A strong debate has continued for a number of years over the magnitude of the ratio of the buoyancy flux b to the rate of production of turbulent kinetic energy from the mean velocity shear. This ratio has traditionally been called the flux Richardson number R_f. In part I of Ivey and Imberger this definition was generalized by broadening the denominator to include all sources and sinks of mechanical turbulent kinetic energy, the net being defined as m. It was shown that for mechanically energized turbulence ($m > 0$, $b > 0$) the magnitude of R_f was completely determined by the magnitude of the overturn Froude Fr_T and the Reynolds Re_T numbers. By contrast, for the penetrative convection case ($b < 0$) R_f was shown to be dependent only on the distance from the source of buoyancy.

In the present contribution, scaling arguments are presented for the magnitudes of Fr_T and Re_T. It is shown that these may vary widely and depend, in the first instance, on the physics of the underlying processes energizing the turbulence. By implication, from Part 1, this means that the ratio of the buoyancy flux b to the net rate of input of mechanical energy m varies between 0 and 0.2 for events where $b > 0$. For events which are energized by a negative buoyancy flux, scaling arguments are used to recast the depth dependence, derived in Part 1, to a dependence on the Reynolds number Re_T.

The magnitude of the pair (Fr_T, Re_T) are then derived directly from temperature microstructure measurements taken in lakes and spanning eight different phenomena: neutral surface layers, penetrative convection, shear layers, diurnal thermoclines, thermals, intrusions, hypolimnetic mixing and boundary mixing. The field data also show large variations in the values of Fr_T and Re_T. Only in the thermocline region of the lake where all mixing events are governed by an inertia–buoyancy balance is Fr_T constant with a value of between 1 and 3.

Thus, both scaling arguments and independent field measurements imply that Fr_T and Re_T may vary greatly throughout a lake. It remains for a more detailed future analysis to verify that the field results and the results from the scaling arguments are compatible for each individual process.

1. Introduction

In Ivey and Imberger (1991, hereafter referred to as Part 1) it was shown that the generalized flux Richardson number, R_f, could be related to the local value of the overturn Froude, Fr_T, and Reynolds, Re_T, numbers of the large scales of motion of the turbulence. This was achieved by reexamining the results from the grid-generated turbulence experiments (Stillinger et al. 1983; Itsweire et al. 1986; Lienhard and Van Atta 1990), grid and shear generated turbulence experiments (Rohr et al. 1988) and from field experiments of penetrative convection (Chou et al. 1986; Mahrt and Lenschow 1976; Guillemet et al. 1983; Kaimal et al. 1976; Imberger 1985a,b; Brubaker 1987; Shay and Gregg 1986).

In brief, it was shown that when $\mathrm{Pr} > 1$ for mechanically energized turbulence ($m > 0$, $b > 0$) R_f is zero at the boundary where the rate of strain Froude number Fr_γ (see Part 1 or section 2) was equal to 3.9, increased to a maximum of 0.20 at Fr_T around one and then decreased again with increasing Fr_T. By contrast, for turbulence energized by a negative buoyancy flux ($b < 0$), the inverse of the flux Richardson number R_f^{-1} decreased from 0.55 at the surface to $-\infty$ near the base of the surface layer.

While most oceanographic measurements (see Gregg 1987 for a review) have yielded $\mathrm{Fr}_T \approx 1$ (see also Peters et al. 1988, Fig. 8), it must be remembered that these measurements were all taken in the thermocline region of the ocean. This would imply that mixing in the ocean always occurred with a value of $R_f = 0.2$ (maximum efficiency of conversion of m to b). Gibson (1980 onwards) has, however, maintained that the reason why field results yielded $\mathrm{Fr}_T \approx 1$ was that the existing measurements greatly undersampled the ocean environment and more active events with $\mathrm{Fr}_T > 1$ have eluded detection.

The variability of the dissipation of turbulent kinetic energy in both stratified lakes and in the ocean has long been recognized as a major obstacle in the measurement of the dissipation itself (Gargett et al. 1984; Imberger and Boashash 1986; Gregg 1987; Gibson 1987b). The difficulty arises because techniques for the measurement of the dissipation, both from the

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